AQA

A-level

Mathematics

MS04 – Statistics 4
Mark scheme

6360
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Version 1.0: Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
$\sqrt{}$or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS04

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 In general, a correct answer (to accuracy required) without units scores full marks

GN4 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN5 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown

GN6 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as 13/47 given as 13:47 or 13:34

GN7 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as 3.21×10^{-3})

Q	Solution	Marks	Total	Comments
1 (a)	$\mathrm{P}(X < 10) = 1 - \mathrm{e}^{-10/16}$ **or** $1 - \mathrm{e}^{-0.625}$	M1		Use of Exp(λ = 1/16 or 0.0625)
	= **<u>0.46 to 0.47</u>**	A1	(2)	AWFW (0.46474)
(b)	$\mathrm{P}(10 < X < 20) =$ $\mathrm{e}^{-0.625} - \mathrm{e}^{-1.25}$ **or** $\left(1 - \mathrm{e}^{-1.25}\right) - \text{(a)}$	B1		Can be implied
	= 0.53526 – 0.28650 = **<u>0.25</u>** **or** = 0.71350 – 0.46474 = **<u>0.25</u>**	B1	(2)	AWRT (0.24876)
(c)	$\mathrm{P}(X \neq 15)$ = **<u>1 or one or unity or 100%</u>**	B1	(1)	CAO
			5	
		Total	**5**	

Q	Solution	Marks	Total	Comments
2 (a)	Differences are: 9 –9 10 8 6 11 –8 7 6 –3 4 10	M1		Attempt at differences
	Mean $\bar{d}$ = **4.25**	B1		CAO; ignore sign
	Sd s_d = **7 to 7.01** **or** σ_d = **6.7 to 6.71** **or** Var s_d^2 = **49.11** **or** σ_d^2 = **45.02**	A1		AWFW (7.00811 or 6.70976) AWRT (49.11364 or 45.02083)
	DF ν = **11**	B1		CAO; can be implied
	CV 95% $\Rightarrow$ t_{11} = **2.2(0)**	B1		AWRT (2.201)
	CI for μ_d is: $4.25 \pm 2.201 \times \dfrac{\left(7\text{ to }7.01 \text{ or } \sqrt{49.11}\right)}{\sqrt{12}}$	M1		Correct use of c's $\bar{d}$ and t/z-value with $s_d/\sqrt{12}$ or $\sigma_d/\sqrt{11}$
	or $4.25 \pm 2.201 \times \dfrac{\left(6.7\text{ to }6.71 \text{ or } \sqrt{45.02}\right)}{\sqrt{11}}$	A1		Fully correct expression
	4.25 ± (4.44 to 4.46) **or** **(–0.21 to –0.19, 8.69 to 8.71)**	A1	**8**	CAO/AWFW (4.45277) Allow reversed signs AWFW
Note	**1** Unpaired CI (using t) $\Rightarrow$ M0 B1 (77.08 – 81.33 = **4.25**) A0 B1 (**22**) B1 (**2.074**) M0 A0 A0 (max of 3 marks)			
(b)	Since **CI includes 0/zero**	Bdep1		Dependent on CI only providing CI **includes zero** Must reference 0/zero
	there is **no evidence**, at 5% level, **to support** Jian's **suspicion**	Bdep1	**2**	OE; Dep on Bdep1
		Total	**10**	

Q	Solution	Marks	Total	Comments
3 (a)	**Diameters** are (approximately) **normally** distributed	B1		
	DF ν = <u>**24**</u>	B1		CAO; can be implied
	CVs 98% $\Rightarrow$ χ^2 = <u>**10.86 and 42.98**</u>	B1		AWRT; either (10.856 and 42.980)
	CLs (variance) are:			(s_d^2 = 0.0029711 s_d = 0.05451)
	ie $\frac{0.071306}{42.980}$ and $\frac{0.071306}{10.856}$	M1		OE; can be implied
	or 0.0016 to 0.0017 and 0.0065 to 0.0066	A1		AWFW; can be implied
	CI (sd) is thus:	M1		Use of square root providing two positive values; can be implied
	<u>**(0.04, 0.08)**</u>	A1	7	AWRT; both (0.04073, 0.08104)
(b)	Since **CI** < **0.10**	Bdep1		Dependent on CI only providing CI is **below 0.10** Must reference 0.10
	there **is evidence**, at 2% level or at 1% level, **of a reduction**	Bdep1	2	OE; Dep on Bdep1
		Total	**9**	

Q	Solution	Marks	Total	Comments
4 (a)	*ThirCars*: Sd = **<u>13.1 to 13.2</u>** **or** Var = **<u>173.6 to 173.7</u>**	B1		AWFW (13.17712) AWFW (173.63636) *Can be scored in (b)*
	NYAutos: Sd = **<u>12.9 to 13</u>** **or** Var = **<u>168.9 to 169</u>**	B1		AWFW (12.99650) AWFW (168.90909)
	H_0: $\sigma_T^2 = \sigma_N^2$ H_1: $\sigma_T^2 \neq \sigma_N^2$	B1		Both
	DF: $\nu_1 = \nu_2 =$ **<u>11</u>**	B1		CAO; can be implied
	CV: $F_{11}^{11}(0.975)$ = **<u>3.47 to 3.48</u>**	B1		AWFW (3.474)
	$F = \dfrac{173.63636}{168.90909}$ **or** $\dfrac{159.1\dot{6}}{154.8\dot{3}}$ = **<u>1.03</u>**	M1 A1		Ratio of two variances AWRT (1.028)
	Thus **accept** that $\sigma_T^2 = \sigma_N^2$	Adep1	**8**	**AG**; dep on *F*-value and CV
(b)	*ThirCars*: mean = **<u>80</u>** *NYAutos*: mean = **<u>34</u>**	B1		CAO; both or $\bar{d} = 46$
	$s_p^2 = \dfrac{11\times173.63636+11\times168.90909}{12+12-2}$	M1		OE; can be implied
	s_p^2 = **<u>171.2 to 171.3</u>** **or** s_p = **<u>13 to 13.1</u>**	A1		AWFW (171.27272) AWFW (13.08712)
	H_0: $\mu_T - \mu_N = 36$ H_1: $\mu_T - \mu_N > \mathbf{36}$	B1 B1		Accept 0 or 3 Must be > 36
	DF: $\nu = 12 + 12 - 2 =$ **<u>22</u>**	B1		CAO; can be implied
	CV: $t_{22}(0.95)$ = **<u>1.71 to 1.72</u>**	B1		AWFW (1.717)
	$t = \dfrac{(80-34)-36}{13.08712\sqrt{\dfrac{1}{12}+\dfrac{1}{12}}}$	M1 M1		Numerator; accept minus (0 or 3) Denominator
	= **<u>1.8 to 1.9</u>**	A1		AWFW (1.87168)
	Thus **evidence**, at 5% level, **to support Maureen's belief**	Adep1	**11**	Dep on *t*-value and CV
		Total	**19**	

Q	Solution	Marks	Total	Comments
5 **(a)(i)**	$\mu = \sum_{x=1}^{\infty} xp(1-p)^{x-1} = \sum_{x=1}^{\infty} xpq^{x-1}$	M1		Ignore limits
	$= p\left(1+2q+3q^2+4q^3+5q^4+...\right)$	M1		Common factor of p with expansion
	$= p(1-q)^{-2} = \frac{p}{p^2} = \frac{1}{p}$	A1	**(3)**	**AG**; fully correct and convincing
(ii)	$\mathrm{E}\left(X\left(X-1\right)\right) = \sum_{x=2}^{\infty} x(x-1)p(1-p)^{x-1}$	(M1)		Only if M1 not scored in (i)
	$= 2pq + 6pq^2 + 12pq^3 + 20pq^4 + 30pq^5 + ...$	M1		Expansion
	$= 2pq\left(1+3q+6q^2+10q^3+15q^4+...\right)$	M1		Common factor of $2pq$
	$= 2pq(1-q)^{-3} = \frac{2pq}{p^3} = \frac{2(1-p)}{p^2}$	A1		**AG**; fully correct and convincing
	Thus $\sigma^2 = \frac{2(1-p)}{p^2} + \frac{1}{p} - \frac{1}{p^2} = \frac{1}{p^2} - \frac{1}{p} = \frac{1-p}{p^2}$	B1	**(4)**	Fully correct and convincing
			7	
(b)	$p = 0.1 \Rightarrow \mu = \underline{\mathbf{10}}$ and $\sigma^2 = \underline{\mathbf{90}}$	B1		CAO both; accept $\sigma = \sqrt{90}$ or 9.49
	Thus require: P(**<u>0.51 or 0</u>** $\le X \le$ **<u>19.49 or 19</u>**)	B1		AWRT; accept P($X \le 19$) alone
	$\mathrm{P}\left(X \le 19\right) = \frac{p\left(1-q^n\right)}{1-q} = 1-q^{19} = 1-0.9^{19}$	M1		Correct attempt at P($X \le x$) or P($X > x$) providing x is an integer value
	= **<u>0.86 to 0.87</u>**	A1	**4**	AWFW (0.864915)
		Total	**11**	

Notes for (a)

(i) Let $S = \left(1+2q+3q^2+4q^3+5q^4+...\right) = \left(1+q+q^2+q^3+q^4+...\right) + \left(q+2q^2+3q^3+4q^4+...\right)$

$$S = \frac{1}{1-q} + qS \quad \Rightarrow \quad S = \frac{1}{(1-q)^2} = \frac{1}{p^2} \quad \Rightarrow \quad \mu = pS = \frac{1}{p}$$

(ii) Let $T = \left(1+3q+6q^2+10q^3+15q^4+...\right) = \left(1+2q+3q^2+4q^3+5q^4+...\right) + \left(q+3q^2+6q^3+10q^4+...\right)$

$$T = S + qT \quad \Rightarrow \quad T = \frac{S}{1-q} = \frac{S}{p} \quad \Rightarrow \quad \mathrm{E}\left(X\left(X-1\right)\right) = 2pqT = \frac{2pqS}{p} = 2q \times \frac{1}{p^2} = \frac{2(1-p)}{p^2}$$

Many other valid solutions are possible and allowable

Q	Solution	Marks	Total	Comments
6	Correct evaluation of $\binom{n}{x}$	M1		At least once
	$\mathrm{P}(R=0)=\mathrm{P}(R=4)=\frac{1\times15}{495}=\frac{3}{99}=\frac{1}{33}$	A1		CAO (accept 0.030 AWRT)
	$\mathrm{P}(R=1)=\mathrm{P}(R=3)=\frac{6\times20}{495}=\frac{24}{99}=\frac{8}{33}$	A1		CAO (accept 0.242 AWRT)
	$\mathrm{P}(R=2)=\frac{15\times15}{495}=\frac{45}{99}=\frac{15}{33}=\frac{5}{11}$	A1		CAO (accept 0.454/5 AWRT)
	H_0: claimed model is appropriate H_1: claimed model is not appropriate	B1		OE At least H_0
	DF: $\nu = 3-1 = \underline{2}$	B1		CAO
	CV: $\chi^2 = \underline{4.6}$	BF1		AWRT; F on ν (4.605) ($\nu = 4 \Rightarrow \chi^2 = 7.779$) ($\nu = 3 \Rightarrow \chi^2 = 6.251$)
		M1		$E = 99 \times p$
		M1		Combining a correct pair of classes at least once
		M1		Attempt at $\sum (O-E)^2 / E$
	χ^2(calc) = $\underline{\mathbf{6.60 \text{ to } 6.70}}$	Adep1		AWRT; dep on M1 M1 M1 (6.65185)
	Evidence, at 10% level, **to suggest claim is not supported**	Adep1	12	OE; dep on CV and χ^2(calc)
		Total	**12**	

r	0	1	2	3	4	Total
Obs	2	18	41	32	6	99
Exp	3	24	45	24	3	99
O		20	41	38		99
E		27	45	27		99
O–E		–7	–4	11		0
(O–E)²/E		1.8148	0.3556	4.4815		

Q	Solution	Marks	Total	Comments
7 (a)	X: Mean = $\underline{np}$ Variance = $\underline{np(1-p)}$	B1		Both means
	Y: Mean = $\underline{3np}$ Variance = $\underline{3np(1-p)}$	B1	2	Both variances; accept $q = 1-p$
(b)	$\mathrm{E}\left(\hat{P}_1\right) = \mathrm{E}\left(\frac{X+Y}{4n}\right) = \frac{np+3np}{4n} = \underline{p}$	M1		One attempted application of E
	$\mathrm{E}\left(\hat{P}_2\right) = \mathrm{E}\left(\frac{1}{2}\left(\frac{X}{n}+\frac{Y}{3n}\right)\right) = \frac{1}{2}\left(\frac{np}{n}+\frac{3np}{3n}\right) = \underline{p}$	A1		Two **correct** applications
	$\mathrm{V}\left(\hat{P}_1\right) = \mathrm{V}\left(\frac{X+Y}{4n}\right) = \frac{npq+3npq}{16n^2} = \underline{\frac{\boldsymbol{p(1-p)}}{\boldsymbol{4n}}}$	M1		One attempted application $(an)^2$ of V
	$\mathrm{V}\left(\hat{P}_2\right) = \mathrm{V}\left(\frac{1}{2}\left(\frac{X}{n}+\frac{Y}{3n}\right)\right) = \frac{1}{4}\left(\frac{npq}{n^2}+\frac{3npq}{9n^2}\right)$ $= \underline{\frac{\boldsymbol{p(1-p)}}{\boldsymbol{3n}}}$	A1		Two **correct** applications
	Consistent as both variances $\to 0$ as $n \to \infty$	AF1		OE; F on both variances having n^{-1}
	Since $\mathrm{V}\left(\hat{P}_1\right) < \mathrm{V}\left(\hat{P}_2\right)$ **or** $\mathrm{RE}\left(\hat{P}_1 : \hat{P}_2\right) = \frac{\mathrm{V}\left(\hat{P}_2\right)}{V\left(\hat{P}_1\right)} = \frac{4}{3} > 1$	Bdep1		Dependent on $\mathrm{V}\left(\hat{P}_1\right)$ and $\mathrm{V}\left(\hat{P}_2\right)$ only providing a numerical ratio from two terms of the form $p(1-p)/an$
	$\Rightarrow$ $\hat{P}_1$ **<u>is more efficient</u>**	Bdep1	7	Dependent on Bdep1
		Total	**9**	